Embedding of Time Series and Data Streams

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DigiCosme, Paris-Saclay

Some information on the University of São Paulo, Brazil

- Founded in 1934 in the State of São Paulo
  - 46 million people
  - 12 million in the city of São Paulo (capital)
    - 21 million in the capital and neighborhoods
- 11 Campi
  - Annual budget 1.2 billion Euros
- 96,364 students
  - 58,823 Undergraduation
  - 14,106 Masters
  - 15,894 PhD
Some information on the University of São Paulo, Brazil

- Students:
  - 52.28% men and 47.72% women

- Staff members
  - 5,844 Professors
  - 14,866 Administrative positions

- World University Ranking of the Times Higher Education

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Overall</th>
<th>Teaching</th>
<th>Research</th>
<th>Citations</th>
<th>Industry Income</th>
<th>International Outlook</th>
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Some Definitions

- What is a data stream?
  - Unbounded sequence of observations collected along time
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  - Unbounded sequence of observations collected along time

- What is a time series?
  - Sequence of observations along time
    - Each observation is typically single dimensional, but that is not a law
      - Ex: The Lorenz System with 3 partial derivatives
    - They can be uniformly or non-uniformly spaced along time
    - It does not assume anything about being unbounded
So let us take advantage of Time Series tools

- From the Statistical branch
  - AR, MA, IMA, ARMA and ARIMA models by Box&Jenkins
  - Auto-Correlation Function to analyze time dependencies
Some Definitions

- So let us take advantage of Time Series tools
  - From the Statistical branch
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- From the Dynamical Systems branch
  - It attempts to find the set of partial derivatives to represent a generating process
  - The generating process is the set of equations that produces a next observations
  - Subdivision:
    - Stochastic approaches
    - Deterministic approaches
Today, we will discuss a little about

Dynamical Systems >> Deterministic approaches

Main results:

In differential topology, there are two Whitney embedding theorems:

- **The strong Whitney embedding theorem** states that any smooth real m-dimensional manifold can be smoothly embedded in the real $2^m$-space, if $m > 0$.

- **The weak Whitney embedding theorem** states that any continuous function from an n-dimensional manifold to an m-dimensional manifold may be approximated by a smooth embedding provided $m > 2^n$. 
Some Definitions

- Takens extended Whitney’s studies to propose an embedding theorem
  - It provides the conditions under which a smooth attractor can be reconstructed from the observations made with a generic function

\[ x(t+(m-1)\tau) = f(x(t), x(t+\tau), x(t+2\tau), \ldots, x(t+(m-2)\tau)) \]

\( \tau \): is the time lag or separation dimension

\( m \): is the embedding dimension

- What is that in practice?
  - Using the Logistic map
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Dynamical Systems >> Deterministic approaches

Main results:


Some Definitions

- They basically define ways of estimating the embedding dimension and the time delay
- Let’s see in practice:
  - Using the Lorenz system
Recently, we came up with a ML approach to estimate both dimensions in conjunction with time-dependent data to provide supervised-learning guarantees, Expert Systems with Applications.

- It builds up some embedding and accesses it using a predictor.

**Most important:** It ensures observations in the reconstructed space are iid, so we can employ supervised learning!!
Conclusions

- What happens if data is not deterministic?
  - We can decompose it, as we will see on the next talk
- Why not taking advantage of time series tools to perform the same with Data Streams?
- Important result:
  - Phase spaces provide iid observations
  - The Statistical Learning Theory is respected and learning is then ensured
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