Measurement-based WCE(R)T estimation: the worst-case is a rare event (?)

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Time critical embedded systems

- Real-time systems
- Cyber-physical systems
Design of a real-time system

Control/Process

Synchronous Models

Processor

Verification

Schedulability

Asynchronous Models

Functional specifications

Implementation

OVSTR, Paris, 15.10.2015

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Overestimated design of real-time systems

• The design of one airplane for the total USA defense budget in 2050 (DARPA 2001)

MIXED CRITICALITY SYSTEMS
Design of mixed criticality systems

- Synchronous Models
- Asynchronous Models
- Control/Processus
- Processor
- Verification
- Schedulability

Functional specifications

Implementation

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Time constraints and schedulability

One processor, fixed priority scheduling

\( \tau_1 \) \( (0, 1, 3, 3) \)

\( \tau_2 \) \( (3, 2, 6, 6) \)

\( \tau_3 \) \( (2, 1, 6, 6) \)

Execution time

Response time

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How do we obtain several values for the execution time of a program?
A probabilistic mixed criticality system

\[ \tau_i \quad (O_i, C_i, T_i, D_i) \]

\[ (O_i, C_i, T_i, D_i) \quad C_i = \begin{pmatrix} 2 & 3 & 5 & 15 \\ 0.9 & 0.07 & 0.02 & 0.01 \end{pmatrix} \]

- Fixed-priority scheduling
- Preemption allowed
- 1 processor

\[ P(P_{si} > D_i) < \varepsilon \]
A probabilistic mixed criticality system (2)

\[ c_2 = \begin{pmatrix} 2 & 4 \\ 0.8 & 0.2 \end{pmatrix} \]
OUTLINE

• Short lesson on probabilities
• Average versus worst case reasoning
• Scheduling algorithms
• Estimation of the probabilistic (worst case) execution time
• One (huge) open problem
• Conclusions
Short lesson on probabilities

• What is a distribution function?

\[ C_i = \begin{pmatrix} 2 & 3 & 4 & 10 \\ 0.5 & 0.3 & 0.15 & 0.05 \end{pmatrix} \]

• What is a 1-cumulative distribution function?

\[ 1{-}\text{CDF} = \begin{pmatrix} 2 & 3 & 4 & 10 \\ 1{-}0.5 & 1{-}0.8 & 0.05 & 0.0 \end{pmatrix} \]
Average versus worst case

What is the impact on an analysis?

- Average number of arrivals within a time interval
  \[ \tau_1 = \begin{pmatrix} 1 & 2 & 4 \\ 0.4 & 0.3 & 0.3 \end{pmatrix}, \text{ for } t_\Delta = 12 \]

- Minimal inter-arrival times between two consecutive arrivals
  \[ \tau_1^* = \begin{pmatrix} 5 & 10 \\ 0.3 & 0.7 \end{pmatrix} \]
• The pET of an instance of a program describes the probability that the execution time of the instance is equal to a given value

\[ C_i^j = \begin{pmatrix} 2 & 3 & 5 & 6 & 105 \\ 0.7 & 0.2 & 0.05 & 0.04 & 0.01 \end{pmatrix} \]
Probabilistic Worst Case Execution Time (pWCET)

• The pWCET of a program describes the probability that the worst-case execution time of the program is equal to a given value
Relation between pWCET and pET

- A safe pWCET is an upper bound on the pETs
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Optimal (task) fixed-priority scheduler

- Rate Monotonic is not optimal

\[ \tau_1 = \left( \begin{pmatrix} 1 \\ 1 \end{pmatrix}, 2, 2, 40\% \right) \quad \tau_2 = \left( \begin{pmatrix} 3 \\ 0.5 \\ 0.5 \end{pmatrix}, 6, 6, 30\% \right) \]
Optimal (task) fixed-priority scheduler

- A feasible task fixed-priority assignment

\[
\tau_1 = \left( \begin{array}{c} 1 \\ 1 \\ 2,2,40% \end{array} \right) \quad \tau_2 = \left( \begin{array}{cc} 3 & 4 \\ 0.5 & 0.5 \end{array} \right), 6,6,30\%
\]
Optimal (task) fixed-priority scheduler (3)

• Theorem (Maxim, 2011)
  The order of higher priority tasks does not have any impact on the probability of missing the deadline of a task
• Audsley reasoning may be proposed
OUTLINE

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How do we get the distributions?

- Static probabilistic timing analysis
  - The pWCET estimate is obtained by convolving the random variables describing worst case execution times of each instruction.

![Convolution Example](image)

\[
\begin{pmatrix}
1 & 2 \\
0.7 & 0.3 \\
\end{pmatrix} \times \begin{pmatrix}
7 \\
1 \\
\end{pmatrix} = \begin{pmatrix}
1+7 & 2+7 \\
1 \cdot 0.7 & 1 \cdot 0.3 \\
\end{pmatrix} = \begin{pmatrix}
8 & 9 \\
0.7 & 0.3 \\
\end{pmatrix}
\]
How do we get the distributions? (2)

- Measurement based probabilistic timing analysis
  - pWCET estimate may be obtained by applying Extreme Value Theory (EVT)
    - Central limit theory for tails
Current status of applying Extreme Value Theory

- Basic EVT: independence hypothesis on execution times
- Dependent EVT: dependence allowed
- Bayesian EVT: learning from history
  - Possible application to CRPD analysis
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One (huge) open problem

A result \[C.al13\]
For a program and a processor it is possible to show that the worst case is a Gumbel, confirming a 14 years conjecture [Edgar and Burns, 2001]


VeCoS 2015, Bucharest
L. Cucu-Grosjean
Validation of a statistical estimation of real-time systems

How do we prove for any program and any processor ????

Design of mixed criticality systems

Complete probabilistic description

Functional specifications

OVSTR, Paris, 15.10.2015  
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Possibles steps (and open problems)

• Worst case probabilistic models
  – Understanding the relations between different design levels
  – Choice of properties to be probabilistically described
  – Proposition of new models

• Time constraints analyses

• Validation and certification of the framework
  – Proposition of a complementary transformation
CONCLUSIONS

• Time critical embedded systems are everywhere
• There is an important bareer to build tomorrow time critical embedded systems
• Mixed criticality systems is a solution
  – Probabilistic Approaches provide good bases to mixed criticality systems
• Proving correct such framework requires an important effort from different communities
Je vous remercie pour votre attention

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One outstanding paper

Scheduling with preemption delays: anomalies and issues

Guillaume Phavorin, Pascal Richard, Joël Goossens, Thomas Chapeaux and Claire Maiza